**Hypothesis Testing:**

Description: Say you flip a coin 10 times and observe only one heads. What would be your null hypothesis and p-value for testing whether the coin is fair or not?

Solution:

First, let's talk about what we can't do:

* We can't set up a 2-sided hypothesis test
* We can't use central limit theorem due to a small n

*H*0 ​: *p*0​=0.5

*H*1 ​: *p*0​<0.5

*p*0​ here is simply the probability of observing heads on a fair coin: 50%.

The null hypothesis is that the coin is fair, and the alternative hypothesis is that the coin is biased: *biased towards tails (note that this a one-sided test):*

*H*0 ​: *p*0​=0.5

*H*1 ​: *p*0​<0.5

Since the sample size here is 10, you cannot apply the Central Limit Theorem and so cannot approximate a binomial using a normal distribution.

The p-value here is the probability of observing the results obtained given that the null hypothesis is true, i.e., under the assumption that the coin is fair. For 10 flips of a coin, there are 2^10 = 1024 possible outcomes, only 10 of which yield 9 tails and one heads.

Hence, the exact probability of the given result is the p-value, which is 10/1024=0.0098102410​=0.0098. Therefore, we can reject the null hypothesis at a 0.05 significance level.